Example of the Third - Order Taylor Method

IVP : $\frac{dy}{dx} = 2 x y$

$y(0) = 2$

$y'' = 2 x y' + 2 y = (4 x^2 + 2) y$

$y^{(3)} = 2 x y'' + 4 y' = (8 x^3 + 12 x) y$

Step size : $h = 0.1$

$\frac{1}{2} h^2 = 0.005$ $\qquad$ $\frac{1}{6} h^3 = \frac{0.001}{6}$

x_0	0	0.1	0.2	0.3	0.4
y_0	2	2.02	2.081411	2.188001	2.346491
y'	0	0.404	0.832564	1.312801	1.877193
y''	4	4.1208	4.495847	5.163683	6.149747
y'''	0	2.44016	5.128596	8.349412	12.464561
$h y'$	0	0.0404	0.083256	0.131280	0.187719
$\frac{1}{2} h^2 y''$	0.02	0.020604	0.022479	0.025818	0.030974
$\frac{1}{3} h^3 y^{(3)}$	0	0.000407	0.000855	0.001392	0.002077
Δy	0.02	0.061411	0.106590	0.158490	0.220770

$$\Delta y = h y' + \frac{1}{2} h^2 y'' + \frac{1}{6} h^3 y^{(3)}$$

x_0	0.5	0.6	0.7	0.8	0.9
y_0	2.567262	2.865492	3.262901	3.790373	4.491905
y ‘	2.567262	3.438062	4.568062	6.064597	8.085430
y “	7.701785	9.857292	12.921088	17.284101	23.537584
y ‘’’	17.970831	25.583111	36.361770	51.912949	74.709370
h y ‘	0.256726	1.343859	0.456806	0.606460	0.808543
$\frac{1}{2} h^2 y''$	0.038509	0.049286	0.064605	0.086421	0.117688
$\frac{1}{3} h^3 y^{(3)}$	0.002995	0.004264	0.006060	0.008562	0.012452
Δy	0.298230	0.397409	0.527472	0.701532	0.938682

x_0	1				
y_0	5.430588				
y ‘					
y “					
y ‘’’					
h y ‘					
$\frac{1}{2} h^2 y''$					
$\frac{1}{3} h^3 y^{(3)}$					
Δy					

Comparison of the results obtained from the Third -
- Order Taylor Method by using various step sizes.

IVP ; $y' = 2 \times y$, $y(0) = 2$

x	h = 0.1 y	h = 0.05 y	h = 0.01 y	h = 0.001 y	Correct y *
0	2	2	2	2	2
0.1	2.020000	2.020088	2.020100	2.020100	2.020100
0.2	2.081411	2.081595	2.081621	2.081622	2.081622
0.3	2.188001	2.188304	2.188348	2.188349	2.188349
0.4	2.346491	2.346953	2.347021	2.347022	2.347022
0.5	2.567262	2.567947	2.568050	2.568051	2.568051
0.6	2.865492	2.866505	2.866658	2.866659	2.866659
0.7	3.262901	3.264402	3.264631	3.264632	3.264632
0.8	3.790373	3.792615	3.792959	3.792962	3.792962
0.9	4.491905	4.495288	4.495811	4.495816	4.496816
1	5.430588	5.435751	5.436557	5.436564	5.436564

* $y = 2\, e^{x^2}$